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A Hybrid System Approach to Contact Stability and Force Control in Robotic Manipulators

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Abstract

Force control and manipulation involving contacts are essentially hybrid control problems because of the inherent switching present in the dynamic behavior when the manipulator comes in contact with and leaves a surface. In this study, the game theoretic approach of hybrid control design is used to synthesize the least restrictive control law for a robotic manipulator to establish and maintain contact with a surface while keeping interaction forces within specified bounds.

1 Introduction and Background

In robotic force control, a problem which often arises is that of "contact instability" which occurs when a rigid manipulator is brought into contact with a very rigid or stiff environment. If the manipulator approaches too quickly, a large impulsive force from the environment acts on the manipulator causing it to "bounce" off. Usually, the manipulator controller will react with a force to bring it back to the surface. If it does so in a way that the approach velocity is as before, the result may be sustained bouncing or contact instability. Obviously, this type of motion could cause damage to the environment or to the manipulator itself and it is desirable to maintain contact after initial impact.

1.1 Literature Review

A review of the relevant literature indicates that contact instability and force control is still a very important and active area of research because it is one of the main obstacles to the ubiquity of robots in everyday life.

In [1], Eppinger and Seering identify non-collocated actuation and sensing as the culprit in causing contact instability. Using a "passive physical equivalent" to model this property, Colgate and Hogan [2] show that this non-collocation results in a fundamental limit on the ability for force control. The primary recommendation of the paper is careful consideration of the hardware design so the controller specifications are within the limits inherent from this problem.

Hogan has experimentally demonstrated, in [3], that impedance controllers can perform stable contact tasks. The primary advantage, when compared to discontinuous hybrid controllers, is in simplicity since the single controller is used for both free and constrained motions. There is no need for control mode switching based on contact sensing. The impedance controller is designed, not to regulate motion or force, but rather to establish a desired dynamic relationship between the manipulator position and the force it exerts on the environment. Kazerooni describes the development of a nonlinear impedance controller for trajectory tracking and experimental results are presented in [4].

The use of kinematically redundant manipulators for alleviating impact problems is described by Walker [5]. The source of the improvement is in the "self-motion" properties of these manipulators and the effectiveness of this approach is dependent on the manipulator configuration.

A number of discontinuous controllers, in which different control laws are applied under different phase conditions, have been proposed. In an ideal hybrid force and position control scheme, a manipulator undergoing free motion is position-controlled while force is monitored. Conversely, when constrained by the environment, the force is controlled while the position is monitored. Any large deviations from expected measurements indicate a change in contact state, resulting in switching of the control and monitoring modes. Paul summarizes some of the practical problems associated with this control methodology in [6]. Using the concept of generalized dynamical systems, Mills demonstrates asymptotic stability of his proposed discontinuous controller [7]. Along with Lokhorst, Mills experimentally verifies that his approach yields the expected and desired results [8]. In [9], Marth *et al* use a hybrid controller in which fi-

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nal contact is established within a predictable time so that one can determine the number of bounces expected, and hence, can establish a sufficient condition for bounceless contact. More recently, Pagilla and Tomizuka (see [10]) describe the design of a discontinuous controller according to three phases of the system (labeled as the inactive, active, and transition phases); stability of this controller is shown via Lyapunov methods.

Force control and manipulation involving contacts are essentially hybrid control problems because of the inherent switching present in the dynamic behavior when the manipulator comes in contact with and leaves a surface. In this paper, we apply the game theoretic approach of hybrid control design to synthesize the least restrictive control law for a robotic manipulator to establish and maintain contact with a surface, while keeping interaction forces within specified bounds.

1.2 Learning from Humans

To solve this problem of contact instability, it is useful to consider how humans perform the same task. Whenever humans use chalk on a board or a pen on paper, contact instability is not observed. Why is this problem so trivial for human beings and yet so difficult for robots? One of the obvious answers is in the compliance inherent in human beings which serves to dampen out motions. By controlling the compliance, or impedance, of the manipulator, we might expect to solve the problem. People are also careful to approach objects at a reasonable speed. Consider a blindfolded individual asked to put his finger on a wall which is at some unknown distance in front of him. This person can be expected to move forward slowly until contact is made rather than break into a sprint towards the wall; anybody foolish enough to try the latter would certainly learn from such a mistake (while recovering from any resulting injuries). Understanding the fundamental dynamics involved when humans carry out rigid tool to rigid surface contact tasks provides much insight into how to avoid contact instability.

2 Game Theory Approach to Hybrid Control Design

The game theory approach to hybrid control design is briefly summarized here. Interested readers may refer to [11], [12], and [13].

In this approach to hybrid control design, the control problem is formulated as a non-cooperative two player zero sum dynamical game. Consider a dynamical system:

$$\dot{x} = f(x, u, d, t) \quad (1)$$

where $x \in X$ is the state, $u(t) \in U$ is the control input, and $d(t) \in D$ is the disturbance. The game is played between the control input and the disturbance as the opponent. Disturbances can be the (unmodeled) environmental disturbances, control inputs from higher level controllers or behaviors of other agents in a multi-agent system. A cost function, $J(x(0), u, d)$, is defined as the objective of the game, a desired behavior that the controlled dynamical system is desired to satisfy. Suppose $u(\cdot)$ is trying to achieve $J \leq C_1$ while $d(\cdot)$ is trying to maximize it. This game is called a zero sum game as the win of one player is the loss of the other. The system is said to admit a saddle solution if there exists a $u^*(t)$ and $d^*(t)$ such that:

$$J(x(0), u^*, d) \leq J(x(0), u^*, d^*) \leq J(x(0), u, d^*) \quad (2)$$

If the system admits a saddle solution, the analysis gives the optimum control strategy $u^*(t)$ and a set of "safe" states in which the control can win the game regardless of the disturbance:

$$S = \{x \in X : J(x, u^*, d^*) \leq C_1\} \quad (3)$$

Given that the initial condition is in the safe set, the least restrictive control law to achieve $J \leq C_1$ is to use u^* when $x(t)$ is at the boundary of S, without any restriction on the control action when $x(t)$ is inside the safe set. Note that this method gives a switching control law although we started with a continuous time system.

3 Controller Design for One-DOF Manipulator

In the one-DOF model of figure 1, the manipulator is modeled with the mass m and the actuator force $\tau \in [-\tau_m, \tau_m]$. The contact is modeled with a nonlinear spring, with the force-position characteristic given by

$$F(x) = \begin{cases} 0, x < 0 \\ kx, x \geq 0 \end{cases} \quad (4)$$

where $k \in [k_{min}, k_{max}]$ is considered as a disturbance. Then, the system is governed by the differential equation

$$m\ddot{x} + k(x)x = \tau(t) \quad (5)$$

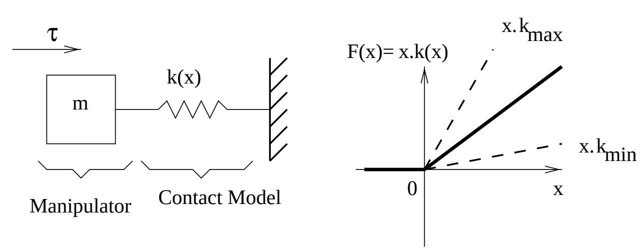

Figure 1: One-DOF Contact Model

The initial conditions are given as the contact force $F(0) = kx(0)$ and velocity $\dot{x}(0)$ at $t = 0$. Contact force is used here instead of position as one usually has sensors to measure the interaction force, but not the relative position.

The problem is formulated as a two player zero sum dynamical game between the players $\tau(t)$ and k, which are, respectively, the control and disturbance inputs, with the objective function being the interaction force F.

We start by considering the in-contact problem, *i.e.*, assume that the manipulator is initially in contact, and we want to maintain contact and avoid applying excessive force. In this case, the safety requirement for the control algorithm is specified in terms of the interaction force as

$$F = kx \in [F_{min}, F_{max}] \tag{6}$$

The lower bound guarantees maintaining contact, and the upper bound avoids excessive interaction force.

3.1 Case 1: $\dot{x}(0) > 0$

In this case, we have the possibility of exceeding the upper limit on the force. First, observe that the maximum force, F_ω, occurs when x is maximum, which is when $\dot{x} = 0$. Here, conservation of energy is used to calculate x_{max}.

The initial energy of the system is:

$$\begin{aligned} KE_0 + PE_0 &= \frac{1}{2}m\dot{x}^2(0) + \frac{1}{2}kx^2(0) && (7) \\ &= \frac{1}{2}m\dot{x}^2(0) + \frac{1}{2}\frac{F^2(0)}{k} && (8) \end{aligned}$$

The energy input is:

$$E_{in} = \int_{x(0)}^{x_{max}} \tau(x)\, dx = \tau(\xi)(x_{max} - x(0)) \tag{9}$$

for some $\xi \in [x(0), x_{max}]$, where at the last step we used the mean value theorem for integrals, assuming $\tau(t)$ is a continuous function in $t \in [0, t_f]$. The final energy is:

$$KE_f + PE_f = 0 + \frac{1}{2}kx_{max}^2 = \frac{1}{2}\frac{F_\omega^2}{k} \tag{10}$$

Solving these equations for F_ω, we find

$$F_\omega = \tau(\xi) + \sqrt{km\dot{x}^2(0) + (\tau(\xi) - F(0))^2} \tag{11}$$

Here, we observe that F_ω is monotone in k and $\tau(\xi)$. Then

$$\begin{aligned} k^* &= k_{max}, && (12) \\ \tau^*(t) &\equiv -\tau_m && (13) \end{aligned}$$

is the saddle solution.

Then, solving

$$F_\omega^* = -\tau_m + \sqrt{k_{max}m\dot{x}^2(0) + (-\tau_m - F(0))^2} \le F_{max} \tag{14}$$

gives, for $F(0) \le F_{max}$

$$\dot{x}(0) \le \sqrt{\frac{(\tau_m + F_{max})^2 - (\tau_m + F(0))^2}{mk_{max}}} \tag{15}$$

as the safe set of initial conditions.

3.2 Case 2: $\dot{x}(0) < 0$

In this case, we have the possibility of losing contact with the surface, *i.e.* causing the interaction force to drop below F_{min}. Similar to case 1, we observe that the minimum force, F_α, occurs when x is minimum, which is when $\dot{x} = 0$. In the analysis we again use conservation of energy.

The initial energy of the system is the same as case 1:

$$\begin{aligned} KE_0 + PE_0 &= \frac{1}{2}m\dot{x}^2(0) + \frac{1}{2}kx^2(0) && (16) \\ &= \frac{1}{2}m\dot{x}^2(0) + \frac{1}{2}\frac{F^2(0)}{k} && (17) \end{aligned}$$

The energy input is:

$$E_{in} = \int_{x(0)}^{x_{min}} \tau(x)\, dx = \tau(\xi)(x_{min} - x(0)) \tag{18}$$

for some $\xi \in [x_{min}, x(0)]$, and the final energy is:

$$KE_f + PE_f = 0 + \frac{1}{2}kx_{min}^2 = \frac{1}{2}\frac{F_\alpha^2}{k} \tag{19}$$

Solving these equations for F_α, we find

$$F_\alpha = \tau(\xi) - \sqrt{km\dot{x}^2(0) + (\tau(\xi) - F(0))^2} \tag{20}$$

which is again monotone in k and $\tau(\xi)$. Then,

$$\begin{aligned} k^* &= k_{max}, && (21) \\ \tau^*(t) &\equiv \tau_m && (22) \end{aligned}$$

is the saddle solution.

Then, solving

$$F_\alpha^* = \tau_m - \sqrt{k_{max}m\dot{x}^2(0) + (\tau_m - F(0))^2} \ge F_{min} \tag{23}$$

gives, for $F(0) \ge F_{min}$ and $\tau_m \ge F_{min}$

$$\dot{x}(0) \ge -\sqrt{\frac{(\tau_m - F_{min})^2 - (\tau_m - F(0))^2}{mk_{max}}} \tag{24}$$

as the safe set of initial conditions.

Putting two conditions together we end up with a "safe" control law such that, as long as

$$\sqrt{\frac{(\tau_m + F_{max})^2 - (\tau_m + F(0))^2}{mk_{max}}} > \dot{x}(0)$$
$$> -\sqrt{\frac{(\tau_m - F_{min})^2 - (\tau_m - F(0))^2}{mk_{max}}} \qquad (25)$$

we are free to use any control action, and whenever we are at the upper (lower) boundary we apply $\tau = -\tau_m$ ($\tau = \tau_m$) to guarantee the force condition of equation (6), for the specified set of disturbances.

The controller can also be characterized for the free space or approach phase using the previous calculations. During the approach phase, as the manipulator has not yet established contact, $F(0) < F_{min}$, therefore the conditions of equation (24) are violated. For free space motion, only the part of the control which deals with F_{max}, given by equation (15), is necessary to avoid excessive interaction forces. To maintain contact, the F_{min} part is activated when $F \geq F_{min}$, which will guarantee no loss of contact.

3.3 Simulation Results

As a demonstration of the use of this control scheme, consider a mass of $m = 1$ kg controlled by an actuator with a maximum input force of $\tau_m = 80$ N. The wall stiffness is taken to be $k_e = 10^4$ N/m and we wish to maintain the contact force within the range $[F_{min}, F_{max}] = [0.5, 50]$ N.

In our first simulation, shown in figure 2, the mass is initially 0.5 m away from the wall and approaching it with a speed of 5 m/s. Because the mass is in free motion, only the upper bound of (25) is activated (the lower bound can only be satisfied during contact). Thus, the actuator applies the maximum negative force, decelerating the mass until it reaches the maximum "approach velocity" which is found by setting $F(0) = 0$ N in equation (15) (here, $v_{approach} = 1.02$ m/s). After reaching this velocity, any control is allowed and the one chosen tracks the maximum approach velocity so that contact is made as quickly as possible; as expected, tracking a constant velocity requires negligible force. As soon as contact is made, the actuator applies the maximum negative force again until the mass is in the "safe" region in which an arbitrary controller can be applied for achieving different criteria. For this simulation, a controller was chosen to provide continuity in the actuation forces between the activation of the different regions (*i.e.,* as the velocity approaches the upper bound, $\tau \to -\tau_m$; similarly, $\tau \to \tau_m$ as the velocity approaches the lower bound). By choosing this type of continuity in the control, the jerk experienced by the mass is reduced. As can be seen in figure 2,

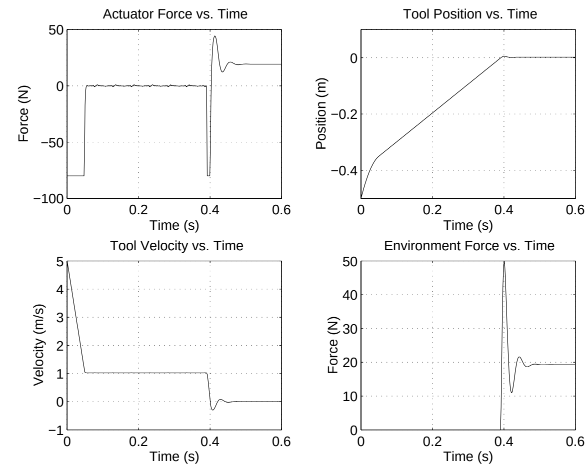

Figure 2: Approach Phase Simulation

the controller performs the desired tasks of bringing the approach velocity to a reasonable value during free motion and maintaining the environment force within the specified bounds during contact, without bouncing.

The second simulation, in figure 3, shows the mass being commanded to track a sinusoidal position trajectory while in contact with the surface. In the safe region, a simple PD controller is applied to the mass. As the position increases, the low-level control is eventually activated with maximum negative force to prevent exceeding the force safety threshold. After being pushed back into the safe set, the PD controller tries to apply a large positive force because of the deviation from the desired trajectory. As can be seen from the actuator force graph, this cycle repeats itself and there is "chattering" in the actuator control. This chattering becomes more severe as the amplitude of the desired trajectory increases. A similar result occurs when the mass attempts to track the trajectory when it leaves the surface. In this case, the maximum positive force is applied to ensure the minimum force is maintained. Once again, the controller achieves the objectives of tracking a desired trajectory within a safe region and bounding the environment forces at the limits of this region. The plot of the actuator force shows that the price to be paid is a higher actuator bandwidth requirement. If this becomes a problem, the designer has the option to choose a more complicated controller in the safe set such that there is continuity between the different regions.

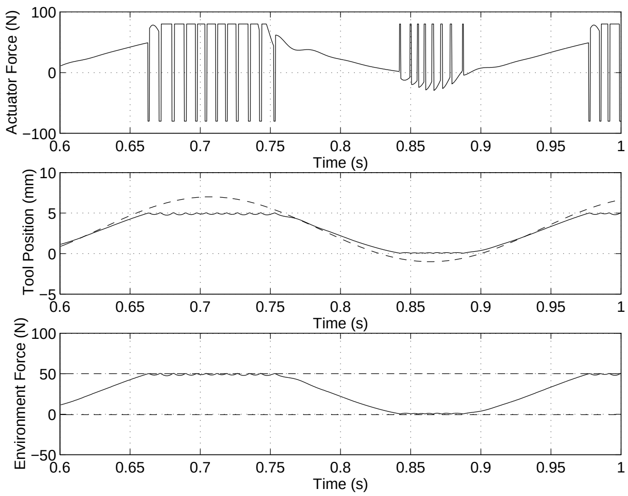

Figure 3: In-Contact Motion Simulation

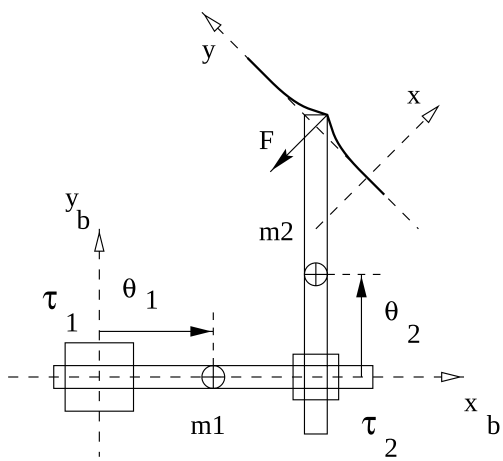

Figure 4: Two-DOF Manipulator Model

4 Extension to Multi-DOF Manipulators

In order to demonstrate the extension of this control scheme to multi-DOF manipulators, consider a two-DOF manipulator with linear joints as an example. Later, we discuss how it can be extended for general manipulators.

The dynamics for the two-DOF manipulator shown in figure 4, in the base coordinate frame (x_b, y_b), are given by:

$$\begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \underline{F} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \qquad (26)$$

where $\underline{F}$ is the interaction force. Assuming we have a frictionless point contact, and modeling the contact force with a spring characterized as in the one-DOF case, $\underline{F}$ will be normal to the surface and proportional to the normal deformation. If we write the equations in a coordinate frame with the x-axis pointing normally into the surface and centered at the rest position of the surface, we get

$$\tilde{M} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} k(x)x \\ 0 \end{bmatrix} = T \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \qquad (27)$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \tilde{M}^{-1} \begin{bmatrix} k(x)x \\ 0 \end{bmatrix} = \tilde{M}^{-1} T \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \qquad (28)$$

Here (x, y) denotes the coordinates of the manipulator tip, and $\tilde{M} = TMT^{-1}$ is the transformed inertia matrix. T is the unitary rotation matrix of the transformation from base coordinates to the world coordinates.

As the interaction force is proportional to the x displacement, we are only interested in the x component of motion:

$$\ddot{x} + \begin{bmatrix} 1 & 0 \end{bmatrix} \tilde{M}^{-1} \begin{bmatrix} k(x)x \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tilde{M}^{-1} T \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \qquad (29)$$

This equation has the form:

$$\ddot{x} + \frac{k(x)x}{m} = \frac{1}{m}\tau \qquad (30)$$

This is exactly the same as the one-DOF case and the same solution applies. For the free space motion, if there is *a priori* information available about the surface normal direction, it can be used to monitor just one direction of velocity. If not, the control can be applied to the magnitude of velocity, so that F_{max} will not be exceeded in any direction.

The general 6 DOF serial chain manipulator has dynamics of the form:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + N(\theta, \dot{\theta}) = \tau \qquad (31)$$

or in the workspace coordinates, (assuming a nonsingular Jacobian):

$$\tilde{M}(\theta)\ddot{x} + \tilde{C}(\theta, \dot{\theta})\dot{x} + \tilde{N}(\theta, \dot{\theta}) = J^{-T}(\theta)\tau \qquad (32)$$

where θ is the vector of generalized joint variables, τ is the vector of generalized joint forces, and $J(\theta)$ is the Jacobian of the inverse kinematics map.

Because of the nonlinear Coriolis terms and the θ dependence of the mass matrix M, direct application of the game theoretic formulation becomes easily intractable for the general case. However, if some bounds on velocity terms can be applied to infer an upper bound for Coriolis and gravitational terms, which is a reasonable assumption as interaction velocities are supposed to be small, we can use feedback linearization to eliminate these nonlinear effects. Assuming the mass matrix and the Jacobian are approximately constant during interaction:

$$\tau = v + J^T(\tilde{C}(\theta, \dot{\theta})\dot{x} + \tilde{N}(\theta, \dot{\theta})) \qquad (33)$$

gives (considering interaction with the environment as well)

$$\tilde{M}\ddot{x} + F(x) = J^{-T}v \tag{34}$$

We use the upper bounds to modify the available actuator torque after feedback linearization, for example:

$$|v_i| \leq \tau_{i,max} - \sigma_{max}(J^T) sup(||\tilde{C}(\theta,\dot{\theta})\dot{x} + \tilde{N}(\theta,\dot{\theta})||) \tag{35}$$

Note that this is a conservative approach, as we are using upper bounds to modify available torques, and also the actuation used for feedback linearization may work against us.

5 Discussion

For hard manipulator to hard surface contact, as is pointed out in [7], the duration of collision is very short, and it is very likely that collisions will end before the controller can act because of the unavoidable computational delays. Given this and the limited bandwidth of the actuation the designed control would be more effective for interaction with a softer environment.

The choice of the controller for the unrestricted region (inside the safe set) is also important for the performance of the controller. A smoothing control law can be used to minimize, if not to completely eliminate, the chattering inherent to the "safe" controller due to its switching nature.

Another application of this control law is to use it as a low level controller to augment other controllers for guaranteed performance. For example, it can be used in a supervisory control algorithm for teleoperation under time delay to guarantee safety.

A more complicated problem currently being studied is the game in which the opponent has the additional ability to control friction. In effect, this results in an uncertainty in the information on the direction of surface normal. The use of this approach to handle control under time delay is also an interesting avenue being pursued, with possible applications for modeling computational delays and the limited bandwidth of actuators.

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